

# Comparison analysis on noise reduction in GPS trajectories simplification

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**Abstract**— Global Positioning Systems (GPS) provide detailed information on the location of an object on the Earth's surface. Among the information they provide are the latitude and longitude coordinates, the time in which it was taken, the direction to which it was traveling, the speed and other parameters. This information is analyzed and stored to support decision making in various sectors. At present, the number of devices equipped with receivers for these systems is increasing considerably as well as the amount of information, which makes the data analysis process difficult and demands greater storage needs. In order to reduce the amount of data to be stored, a set of algorithms for GPS data simplification, that carry out the spatial-temporal analysis of the data are proposed; however, the nature of the data is not taken into account. In the present investigation a comparison of different algorithms for GPS data simplification is carried, taking into account the noisy nature of the same ones. In order to reduce the noise present in vehicular trajectories, the Kalman filter is selected because it predicts the next state starting from the previous state, taking into account the dynamics of movement of objects. As a result, a comparison is obtained among some of the algorithms for vehicular GPS trajectories simplification that constitute the base of the formulation of more complex algorithms before carrying out the filtering and after the data filtering is carried out.

**Keywords**— GPS vehicular trajectory data, Kalman Filter, GPS simplification algorithm, GPS data reduction.

## I. INTRODUCTION

The evolution of positioning technologies through global positioning satellites and 3G and 4G cellular networks has increased the amount of data generated by an increasing number of devices. Nowadays, the margin of error in GPS data is not only associated with the triangulation of satellites with respect to the device or noise in the sensor, measurement errors and others, but is also given by the position calculated by the telephone towers, which poses new challenges.

Spatial paths are inaccurate due to different conditions [1]. Correct filtering of this type of signal reduces the noise and potentially the error in the measurement of this type of signal. Currently, GPS data are widely used in intelligent transportation systems and in the guidance of autonomous vehicles. On the other hand, the efficient storage of vehicle trajectory data provides a set of historical data for urban analysis, traffic and congestion in main arteries and roads. The amount of data obtained in a large city per day and their accuracy are two impact problems in the field of GPS data processing.

GPS data are used by various systems for georeferencing, intelligent transportation systems, trajectories calculation, urban planning or localization of autonomous vehicles, just to mention some application examples nowadays. The efficiency and precision of these GPS data is of vital importance to guide autonomous vehicles, calculate routes accurately and other multiple solutions to problems of modern life.

The transmission of vehicular GPS data associated with known trajectories and the limitations of communication between vehicles and the network itself are other active research topics in this field. Vehicle communication and data transmission between vehicles or between vehicles and an intelligent transportation system is also affected by the amount of data transmitted. The use of historical data makes it possible to find solutions to short roads or the tracing of new routes, so storing data efficiently for transmission is a priority. The quality of data associated with noise levels and the amount of data to store and transmit are two of the issues addressed in this work.

To mitigate noise problems, the literature reports the use of Kalman filter as the optimal solution for predicting the next state starting from the current state in this type of signals [2]. Several authors have designed filters for noise reduction in GPS trajectories using data from both telephone networks and GPS receivers. Related to data simplification, several algorithms have been proposed that use different techniques to reduce the number of points needed in the representation of a trajectory. These methods of GPS trajectories simplification perform the elimination of redundant points but do not take into account the origin of the data or its nature.

The noisy nature of GPS data negatively influences the data simplification. Points located meters away at a different angle can be taken by the wrong criterion in the simplification process, which translates into loss of sensitive information or use of redundant information within a simplified trajectory. A quality data simplification process improves the effectiveness of storage and makes it possible to reconstruct the simplified trajectory to use it in georeferencing systems, as a guide for autonomous vehicles or in urban planning, among others.

This research compares different methods of GPS trajectories simplification before and after data filtering in order to evaluate the impact of filtering on the simplification process. section 2 analyses the theoretical aspects related to the most used noise reduction filters in the bibliography for the problem of noise reduction in GPS trajectories, as well as the methods for GPS trajectories simplification. In section 3 the model used for the implementation of Kalman filters and the simplification methods to be analyzed are described, and in section 4 the

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analysis of the results obtained after the experimentation is carried out.

## II. BACKGROUND WORK

The problem of GPS trajectories simplification has been approached by different authors [3]-[8] as a specific problem, assuming that the data have been filtered or are accurately taken, without taking into account the inaccuracies derived from the data capture process. Other authors, on the other hand, have dealt with the issue of data filtering, mostly using the Kalman filter [9]-[11] to then process the data. In the analysis carried out, only [12] shows work associated with the application of Kalman filters and the GPS trajectories simplification process. In this work, the results of performing data filtering using Kalman and then using a hybrid method for GPS trajectories and segments simplification are presented. Although this work does not validate the improvement that the data undergoes when filtered, it does perform an analysis on the improvement in the segmentation process presented by the introduction of the Kalman filter for data pre-processing.

These methods [12] divides data into trajectories that represent trips with a starting point, a set of intermediate points, and an end point. It then filters the data to decrease the level of noise present in the paths using the Kalman filter. Once the data has been filtered, the stationary points are detected and denoted in order to know the segments in which the driver goes at a speed below a threshold. Finally, the spatial-temporal analysis of the filtered data is carried out using a combination of Douglas Peucker algorithm and Sliding Windows. The above results demonstrate the feasibility of adding Kalman filter logic to the simplification algorithm as part of this process.

### A. GPS trajectory filtering

The Kalman filter algorithm consists of five main processes which are listed below:

- Prediction of the next system state
- A priori covariance updating
- Kalman gain calculation
- Estimation of the current state
- A posteriori covariance updating

In the first place, predict the next state of the system using the system process model. Assuming that the system is now  $k$ , according to the system model, it is possible to predict the current state based on the last state. This is described in equation 1

$$X(k|k-1) = AX(k-1|k-1) + BU(k) \quad (1)$$

Where  $X(k|k-1)$  represents the result of the prediction,  $X(k-1|k-1)$  represents the optimum state of the last prediction,  $U(k)$  represents the current control value,  $A$  y  $B$  are parameters of the modeled system. In the present investigation these parameters are adjusted to 1,  $X(k-1|k-1)$  represents the last location in the form of  $(x, y)$  coordinates and  $U(k)$  the distance of the interval, which in the present investigation represents the distance of the trajectory.

Then, the covariance corresponding to  $X(k|k-1)$  is updated using expression 2:

$$P(k|k-1) = AP(k-1|k-1)A^T + Q \quad (2)$$

Where  $P(k-1|k-1)$  is the covariance corresponding to  $X(k-1|k-1)$ ,  $A^T$  represents the transposed matrix of  $A$  and  $Q$  is the covariance of the system procedure. This research uses the same value of  $Q$  used in the reference investigation, so  $Q = 10$ .

Once these values are obtained the calculation of the Kalman gain, denoted as  $Kg(k)$ , is performed, which is described in equation 3:

$$Kg(k) = P(k|k-1)H^T / (HP(k|k-1)H^T + R) \quad (3)$$

Where  $H^T$  represents the transposed matrix of  $H$  and  $R$  is the covariance of the noise measurement.

Then, these results are combined with the predictive value and the measurement value to obtain the optimal estimates of the current state as described in equation 4:

$$X(k|k) = X(k|k-1) + Kg(k)(Z(k) - HX(k|k-1)) \quad (4)$$

Where  $Z(k)$  represents the current measurement value and  $H$  is set to 1. Finally, the covariance corresponding to  $X(k|k)$  is updated using equation 5:

$$P(k|k) = (1 - Kg(k)H)P(k|k-1) \quad (5)$$

Where 1 represents the identity matrix and  $P(k|k)$  corresponds to  $P(k-1|k-1)$  of equation 2 when the system enters at time  $k+1$ . In this way the Kalman filter continues until the end of the process.

Several authors have analyzed the GPS trajectories filtering process from the characteristics of the data using this filter because it estimates the optimal prediction value by properly mixing the predictions made by a mathematical model with the values measured by the sensors. In [13] Kalman filters are used to reduce noise in trajectories taken with low-cost sensors and improve accuracy. A tractor is used as the vehicle for the measurements and a receiving device is placed at the gravity center of the vehicle. The equations that guide motion for this vehicle are formulated and replaced in Kalman filters for more accurate results.

In [14] exposes the role of Kalman filters in GPS error modeling. This paper discusses the formulation of Kalman filters for GPS data filtering and demonstrates a significant improvement by analyzing filtered and unfiltered data, showing that differences can be softened using this filter.

Other works describe the use of the Kalman filter for GPS data filtering in order to soften trajectories and reduce the noise they present. In the bibliography, depending on the domain of the problem, standard Kalman filters or any variant of them are used. In the case of the present investigation the standard filters are used for data filtering because the data are known beforehand and the problem is modeled in a linear way.

## B. GPS trajectory data compression methods

The growing development of technology and the use of new devices equipped with sensors to determine their location generate a high volume of data. The compression of GPS trajectory data is a field of research that arises from the need to optimize the use of disk space occupied by GPS data. Currently, trajectory simplification methods are used to reduce the number of points it contains.

Among the trajectory simplification methods described in the bibliography are:

1. Line simplification
2. Douglas Peucker
3. Top Down Time Ratio
4. Visvalingam Whyatt

These algorithms perform spatial or space-time analysis of the data. Line simplification and Douglas-Peucker methods only perform the spatial analysis, while the others perform the spatial and temporal analysis of the data to simplify the redundant points.

Douglas-Peucker (DP) is a GPS trajectory compression algorithm that uses the top-down method for data analysis. It is used to remove a series of line segments from a curve, which reduces the amount of data present in a GPS trajectory [15]. It is a line generalization algorithm that recursively selects points from the original series of GPS trajectory points [3] [16]. It implements the divide and conquer strategy and it is composed of several steps [17]:

1. The first and last node of a polygonal chain are taken as endpoints of a line.
2. For all intermediate points, distances less than this line are determined. If it is greater than any distance, the polygonal chain is separated with the point having the greater distance, forming two new segments.
3. The new segments formed are analyzed using the same procedure.
4. It ends when no punctual line distance is exceeded.

Top Down Time Ratio [18] is a modification to the Douglas-Peucker algorithm where the time variable is added to perform the space-time processing of the data. For this, the coordinates of the  $P_i$  point in time are calculated by means of the radius of two-time intervals. The distance function is modified based on the use of time and is called synchronous Euclidean distance.

The Visvalingam-Whyatt simplification algorithm [19] [20] uses the concept of effective area, which is defined as the area of the triangle formed by a point and its two neighbors. The algorithm takes a polyline  $P$  as the sequence of points  $\{p_1, \dots, p_n\}$ , and the spatial displacement error is defined by the user. For each set of three consecutive points  $\{p_{i-1}, p_i, p_{i+1}\}$  a triangle is formed, this being the effective area. Iteratively the point  $p_i$  that produces the smallest displacement of the area is selected to form an approximation. This process stops when the effective area is greater than  $\epsilon$ . This algorithm presents a hybrid approach between online and batch. It is designed to

preserve spatial-temporal information, direction and speed in a trace.

These GPS trajectory simplification methods perform the elimination of points that are considered less important within the trajectory. The elimination must be carried out under the precept of minimum affection to the original trajectory, in such a way that the trajectory can be represented even in the absence of all the points. Each method has been tested in different databases and its limitations have been recorded. In the present research these methods are selected because they are the basis of more complex methods for the simplification of points and because they share a common characteristic, none of the methods used for GPS trajectories simplification takes into consideration the noisy nature of the data.

## III. NOISE REDUCTION AND TRAJECTORIES SIMPLIFICATION

In this research, a comprehensive search is conducted to find previous works that perform the pre-processing of trajectories before the simplification step. However, it was found only one referent that performs this process [12]. In that research the analysis is carried out only with a hybrid algorithm that includes Douglas-Peucker and Sliding Windows algorithms.

The objective of this work is to make the comparison, before the filtering process and after the filtering process, of various algorithms for GPS trajectories simplification. The Kalman filter is used for the filtering process as described below:

Initially, a model is built, closely related to the trajectory to be analyzed in order to adjust the filter. The mathematical definition of the Kalman filter used is based on the one defined in [1], [12] because it makes use of the mathematical model of a 4-wheel vehicle.

The modeling of the movement problem is defined in this research by the movement equations 6 and 7:

$$x_k = x_{k-1} + v_{k-1} * \Delta t * \cos\theta_{k-1} \quad (6)$$

$$y_k = y_{k-1} + v_{k-1} * \Delta t * \sin\theta_{k-1} \quad (7)$$

For the modeling of the problem to be solved, the type of data and the specific conditions of the data were taken into account. The data are known beforehand and the GPS trajectories are basically composed of: speed, time, position in the form of (x, y) coordinates and the movement direction. Once the initial moment has been established, the problem properly modelled and the movement equations established, the data are filtered using the Kalman filter described in section 2. The states through which data transition are:

1. Prediction of the next system state
2. A priori covariance updating
3. Kalman gain calculation
4. Estimation of the current state
5. A posteriori covariance updating

For the application of this filter in the present investigation, the input data are defined as initial state or  $X_k$  state variables.

$$X_k = \begin{bmatrix} x \\ \dot{x} \\ y \\ \dot{y} \end{bmatrix}$$

As covariance matrix, the C matrix is defined as:

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The state matrix or ME transition matrix is defined, which contains part of the movement equations that have been developed to model the problem:

$$ME = \begin{bmatrix} 1 & 0 & 0 & \Delta t * \cos\theta_{k-1} \\ 0 & 1 & 0 & \Delta t * \sin\theta_{k-1} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The covariance matrix of the observed noise or observation matrix called R is defined.

$$R = \begin{bmatrix} co & 0 & 0 & 0 \\ 0 & co & 0 & 0 \\ 0 & 0 & co & 0 \\ 0 & 0 & 0 & co \end{bmatrix}$$

Where *co* represents the covariance of the observations. This covariance is calculated using the expression 8

$$co = \frac{\sum(x_i - x_{prom})(y_i - y_{prom})}{n-1} \quad (8)$$

So, the prediction state is represented as shown in expression 9:

$$X_{k+1} = ME * X_k \quad (9)$$

Once the data has been filtered using Kalman's filter logic, the Douglas-Peucker, Top Down Time Ratio and Visvalingam-Whyatt algorithms are implemented to carry out the comparison. These three algorithms represent the spatial and spatial-temporal analysis of the data performed on most of the algorithms proposed in the bibliography. The Douglas-Peucker algorithm is selected in a particular way because it is one of the first proposed algorithms and it is taken as a reference in most of the associated works on this matter.

As a methodology it is proposed to carry out the filtering and simplification process to individual GPS trajectories and not to databases as a whole. In this way, comparable data can be obtained with what would be the use of this research in practice, where a trajectory once completed is simplified and stored to be used as a reference in future analyses.

#### IV. RESULTS AND DISCUSSION

The use of a filter to reduce the noise present in the trajectories is one of the topics to be considered in order to propose more efficient algorithms for GPS trajectories compression. Noise is one of the factors that limit the efficiency of the trajectory simplification process by incorrectly selecting

a set of noisy points. The present investigation aims to test this construct and three experiments are designed for this purpose.

The first experiment aims to determine the amount of noise present in GPS trajectory data. The second experiment aims to determine the number of points that are removed by the different algorithms when the data has not yet been filtered. The third experiment aims to determine the number of points that are removed by the different algorithms when the data have already been filtered and thus establish a comparison.

For the experimentation process, a California taxi database called Mobile Century is used. The database consists of 914684 points arranged in 1589 trajectories. The data used for the analysis are the trajectory identifier, latitude, longitude and time. With these values other important values can be obtained in the formulation of improvements to the filter or to the modeling of the problem in certain specific situations.

The first experiment receives the allegedly noisy data as input and the filtered data are obtained as output. During the initialization of the Kalman filter, the equations that characterize the movement for each object are determined and introduced in the model. In this way, the result of the application of the filter softens the trajectories according to the position, the moment of time and the angle in which the GPS point is taken. Once filtered the entire database, divided by trajectories, it is obtained that 98.6 percent of the data present changes in the values of latitude, longitude or both, regarding the original data. This change in the data is due to the fact that the Kalman filter estimates the optimal prediction value, adequately mixing the predictions made by the mathematical model described in the previous sections, with the values measured by the sensors and takes into account the dynamics of objects movement.

These results indicate that data filtering can correct the noise present in this type of signal. In the bibliography, the main causes associated with the noise present in the trajectories, such as inertial sensor errors, have been described, which can be classified mainly as first-order biases, scale factors and misalignments.

The objective of the second experiment is to determine not only the quantity but also the points eliminated during the simplification process using a specific algorithm. This simplification process is carried out before the data is filtered and therefore potentially noisy data is considered to be simplified.

Table 1 shows the results obtained from the execution of the simplification algorithms considering that the threshold defined for the selection of points in TD-TR is 500 meters, the epsilon value defined for Douglas-Peucker is 0.02 and for Visvalingam-Whyatt a threshold value is used for the effective area defined in 0.036.

The results shown correspond to the amount of GPS data remaining after applying the simplification algorithms.

TABLE I. AMOUNT OF POINTS REMAINING AFTER APPLYING THE SIMPLIFICATION ALGORITHMS

Amount of Points of Original GPS Trajectory	Remaining Point After Simplification Algorithms Applied		
	Douglas-Peucker	Top Down Time Ratio	Visvalingam-Whyatt
7607	611	254	291
6147	413	212	239
6708	479	208	261
6797	1091	280	300
8067	467	244	314

The third experiment assumes that the data has been correctly filtered using Kalman's filter logic. It receives as input data the filtered data and as output data the simplified data are obtained. Table 2 shows the results expressed in number of simplified points for each algorithm.

TABLE II. AMOUNT OF POINTS SIMPLIFIED BY EACH ALGORITHM.

Amount of Points of Original GPS Trajectory	Remaining Point After Simplification Algorithms Applied		
	Douglas-Peucker	Top Down Time Ratio	Visvalingam-Whyatt
7607	590	254	289
6147	390	208	238
6708	420	212	260
6797	773	271	290
8067	456	248	265

As can be seen in the results shown in table 2, the simplification process is improved by applying a noise reduction filter to the trajectories. In almost all cases the result of the experiment with filtered data is better than the result of the experiment with unfiltered data. The amount of simplified points is higher and the quality of simplified points increases when comparing results point-to-point. This shows that the deviation of a few meters in latitude, longitude or both on a GPS point, significantly influences its selection by the simplification algorithm.

A measurement of the amount of time, in milliseconds, that the simplification process takes was performed, obtaining an average of 0.39 milliseconds while for the filtering process 20 milliseconds were obtained. These values were obtained in a PC core i5 of 4 physical cores with 8 GB of RAM memory. These times can be reduced considerably if optimizations are made to the implementation and executed in a server with better hardware performance.

Although the results obtained from the simplification experiment with filtered data are better than those obtained from the experiment with unfiltered data, it is considered that in this field it is possible to analyze the logic of other algorithms that

allow to compress the resulting data. In this way, the minimum data to reconstruct a trajectory would be maintained while reducing the disk space needed to store this data. Process performance is increased by including data filtering logic in the GPS trajectory simplification process due to the operations performed. However, considering that this process is carried out offline and that it improves the efficiency of the process, it is considered that it does not negatively influence the results.

## V. CONCLUSIONS AND FUTURE WORK

In the present work, the logic of Kalman filters was implemented alongside Douglas-Peucker, TDTR and Visvalingam-Whyatt GPS trajectory simplification algorithms. The analysis of the bibliography showed that the Kalman filter is the optimal filter for noise reduction present in the trajectories. The implementation of Kalman's filtering logic allowed to decrease the noise present in the trajectories. The application of simplification algorithms allowed the elimination of points that are not significant or necessary to correctly represent a trajectory, which decreases the amount of disk space needed to store them. The established comparison of the simplification process with and without data filtering shows that the filtered data performs the simplification process in a more efficient manner, which facilitates its storage and improves data transmission between autonomous vehicles and the intelligent vehicle system. In this work was only analyzed the concept of data simplification, having as future work the application of an algorithm for the verification of the remaining points and an algorithm for data compression without loss, bearing in mind that it is desired to reduce to the maximum the space used for data storage.

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